Risk and the Misallocation of Human Capital

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Abstract

When workers are risk-averse and shocks to earnings are uninsurable, competitive markets allocate too few workers to jobs with high earnings uncertainty. Using a general equilibrium Roy model with incomplete markets we show that occupational risk is partially compensated in a *laissez faire* economy. However, risky occupations are inefficiently small compared to the social planner allocation or a complete markets economy; hence talent is misallocated. We obtain analytical expressions for the compensation for risk in the labor market, as well as for the aggregate level of human capital and output. Misallocation is positively related to the correlation between a worker's abilities in different occupations. In our quantitative analysis we find that only market incompleteness can generate output and welfare losses in the order of one percent of GDP, permanently.

Key words: Risk Premium, Human Capital, Occupations, Labor Markets, Misallocation, Frechet, Roy Model.

JEL Classifications: E21 · D91 · J31.

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1 Introduction

Human capital misallocation lowers productivity. Occupation or industry-specific human capital is an important feature of labor markets. For example many technical, medical and legal occupations require knowledge in a narrowly defined field. It is rarely possible to work in such occupations without first obtaining occupation-specific skills and credentials through specialized training. At the same time, due to technological progress, international trade or urbanization, workers in certain occupations are subject to permanent earnings shocks that are hard to predict when deciding the set of skills to acquire. The fear of high potential losses arises because private insurance markets to hedge against these shocks are missing. A pervasive feature of modern economies is the occurrence of sizable negative shocks to certain occupations or industries that are difficult to predict and to insure against. These shocks displace workers that are heavily invested in occupation- or industry-specific human capital.

In this paper we study how uninsurable permanent risk to a worker's human capital shapes the aggregate allocation of talent. Through the prism of a Roy model we show that risk is partially compensated, and as a result talent is misallocated in a *laissez faire* competitive equilibrium. Risky occupations are inefficiently small as risk averse workers avoid them when insurance opportunities are absent. In our quantitative analysis we study cases in which shocks to workers' human capital are either caused by policies (i.e. trade reforms) or by the process of economic development (technological progress or urbanization). We find that the misallocation caused *only* by market incompleteness produces permanent output losses of around 0.6% of output. Our results shed new light into the cost of market incompleteness. They can inform policymakers when designing policies aimed at providing earnings or unemployment insurance for workers.

Our general equilibrium Roy model features a labor market where workers selfselect into an occupation or industry based on their comparative and absolute advantages.[1] We assume that workers are risk-averse and human capital (for example acquired through specialized university training) is specific to an occupation or industry. Workers' occupational choices determine both the level of output and the wage distribution in the economy. We compare the production efficiency in competitive equilibrium to an unconstrained planning problem which yields maximal output.

Our model features two occupations (without loss of generality) and the choice of a career is based on two factors: (i) a worker's inherent talent in a given occupation, and (ii) an occupation's earnings uncertainty, measured by the variance of permanent shocks to earnings. The inherent talents of workers are modeled as draws from a Frèchet distribution. We allow for abilities to be correlated, which provides us with a tractable way of distinguishing between comparative and absolute advantages. One extreme case is that of perfectly correlated draws in which a worker's ability is the same across occupations (absolute advantage). The other extreme would be the case of independent draws (comparative advantage). The model's tractability allows us to obtain closed-form solutions for various outcomes of interest such as the allocation of workers, output, and the wage and earnings premia.[2] Thus, a key advantage of our framework is that it is tractable, and thus the mechanics of how the interplay between ability and risk affects the allocations and output are transparent.

We measure misallocation by comparing output in a competitive equilibrium to output achieved by a social planner. The planner allocates workers across occupations based on their abilities in order to maximize output. Of course, the planner does not observe the shocks that workers receive once they have chosen an occupation. However, she can allocate consumption across workers after shocks are realized. The

[1]Except in the quantitative analysis, in the remainder of the paper we use the terms industry and occupation interchangeably.

[2]By a wage or an earnings premium we refer to the wage or earnings differential between the risky and the safe occupation.

planner's allocation is identical to that obtained in a competitive equilibrium with risk-neutral workers. Although risk is compensated in the competitive equilibrium — riskier occupations pay more — the planner allocates more workers to riskier occupations than the competitive equilibrium does, resulting in higher output. In a competitive equilibrium, the link between the marginal product of labor and the wage prevents the size of risky occupations to grow to the efficient level. At the efficient level, wages are too low to compensate for the extra risk borne.

As expected, misallocation is more severe the higher the workers' risk aversion. As risk aversion rises, entering the risky industry is less desirable and thus higher risk aversion exacerbates the costs of market incompleteness. We also find that the degree of misallocation is negatively related to the degree of comparative advantage. Independent draws (the extreme case of comparative advantage) imply a higher degree of selection because good abilities can only be used in one occupation. When the dependence is low for both abilities there is a higher likelihood that the worker has high ability in at least one occupation. The higher selection – i.e. the higher ability by occupation – implies a better buffer against risk. Therefore, the absence of insurance markets matters less. As an additional result, we also provide a simple tax scheme that restores the planner's allocation.

Our quantitative analysis focuses on three questions that have received attention in the literature. We begin by calibrating the model to US data on earnings by industry. We use estimates of the variance of permanent shocks to earnings by industry and pick values for the rest of the parameters to match moments of the 2001 wave of Survey of Income and Participation Program. The earnings premium in the data is around 7% (after controlling for observables like education and age) which renders a risk aversion parameter of 2.9. We find that the maximum output loss due exclusively to market incompleteness can be as high as 0.6% permanently.

We also use our model to quantify the output losses associated with trade reforms. For this purpose, we make use of a number of studies that document a positive relationship between the degree of import penetration and trade exposure of an industry with the volatility of workers' earnings. We take as given the increase in import penetration of the US manufacturing sector in the period 1991-2009. This rise in import penetration caused a reallocation of manufacturing workers. In light of our model, this reallocation is not efficient and occurs because the increase in risk due to trade openness makes the tradable sector less attractive. The increase in misallocation only due to the increase in risk in that period can plausibly be as large as 0.7 percentage points. The corresponding decrease of the manufacturing sector the model predicts can be as large as 4 percentage points (a third of that observed in the US). As in our theory, we can easily solve for the tax and subsidy scheme that recovers the first-best allocation.

We finally use our model to quantify the importance of market incompleteness in explaining the observed low productivity levels and the larger share of rural workers in poor countries. In light of our model, the existence of informal insurance markets in the rural areas, everything else constant, makes the rural occupations more attractive to workers. Therefore, from the perspective of a social planner, a larger than desirable proportion of workers choose rural occupations. In addition, the more able workers are the ones who choose an urban occupation. We use micro data from Ethiopia to calibrate our model and to measure the degree of misallocation of workers between rural and urban areas. In our data, 72% of Ethiopian workers are rural and the average earnings of urban workers are 25% higher than rural workers. At the same time, measured volatility of earnings is higher in the urban areas. We estimate output losses that can be as high as a quarter of a percentage point permanently.

1.1 Related Literature

The paper connects several strands of the literature in macroeconomics, labor economics and development economics. First it relates to the macroeconomics literature on misallocation and development. As has been studied in many important papers (see e.g. Hsieh and Klenow (2009), Restuccia and Rogerson (2013), Lagakos and Waugh (2013), Lagakos, Mobarak, and Waugh (2018), Vollrath (2009), Midrigan and Xu (2014), Guner, Ventura, and Yi (2008)) misallocation of factors of production across firms, sectors or regions within an economy are important to explain cross-country productivity differences. However, with some exceptions (see for example Vollrath (2014) and Hsieh, Hurst, Jones, and Klenow (2019), Buera, Kaboski, and Shin (2011), Bhattacharya, Guner, and Ventura (2013)) the misallocation of human capital has received much less attention. In our case we focus on one particular friction and thus a source of misallocation: the inefficiency of the competitive equilibrium allocation caused by incomplete markets. On one hand this focus allows us to analyze the consequences of specific friction whose existence is hardly debatable. On the other hand we abstract from other important barriers to the allocation of workers to occupations and thus our results regarding the output losses may seem smaller than the ones reported for example in Hsieh, Hurst, Jones, and Klenow (2019). Furthermore, our analysis does not focus on some specific human capital or occupation since it includes all the occupations and is flexible enough to incorporate other aspects of the occupational or industry choice done by workers.

Our focus on the effects of the lack of insurance markets on the allocation of workers, motivates our quantitative analysis that uses Ethiopian data and thus relates our paper with the literature on development economics that highlights the importance of differential access to insurance markets between rural and urban areas and its effect of rural productivity on aggregate productivity. Important contributions in this area are Harris and Todaro (1970), Townsend (1994), Udry (1994), Munshi and Rosenzweig (2016), Donovan (2020) and Morten (2019). We do not focus on migration but on measuring the effect of risk in the allocation of workers and measure the output loss associated with the absence of insurance markets in urban areas. Although there is no consensus in the literature (see Lagakos (2020) for an excellent review), our model can contribute to the discussion on the importance of market incompleteness in explaining the observed low productivity levels and the larger share of rural workers in poor countries.

Our theoretical approach uses the important insights of Roy (1951) and model worker's occupational choice under uncertainty. Thus, it connects to models of occupational choice used in macroeconomics and labor economics. Examples are Kambourov and Manovskii (2008, 2009), Jovanovic (1979), Miller (1984), Papageorgiou (2014), and Lopes de Melo and Papageorgiou (2016). We focus on the interplay between comparative advantages and risk in shaping worker's occupational choice and thus we complement their findings as well as the ones present in Cubas and Silos (2017, 2020), Silos and Smith (2015), Hawkins and Mustre del Rio (2012), Dillon (2016), and Neumuller (2015). The main difference with these papers is that we abstract from modeling the career dynamics so we can obtain closed form solutions and a better characterization of the elements that affect the misallocation of human capital.

Finally, in terms of our quantitative analyses, our application to Ethiopia relates our work to Cai (2016) who also uses a Roy model with incomplete markets to measure the cross-country differences in agricultural productivity differences. We also think our application to trade reforms provides new insights to the literature trying to understand the effects of trade reforms on labor markets. Our framework does not incorporate international trade but is flexible enough to measure the output losses associated with trade reforms when workers who are exposed to import competition are unable to insurance against permanent shocks to their earnings. Thus, our work is related to the work of Lyon and Waugh (2018), Lee (2020) and Traiberman (2019).

2 Model

The economy is populated by a continuum of workers of total mass equal to one who live for one period. They are endowed with a unit of time they inelastically supply as labor. That unit of labor can be supplied in either of two occupations (labeled occupations 1 and 2).[3] Workers value the consumption of a final good produced according to the following CES technology.

$$Y = [\theta N_1^{\nu} + (1-\theta) N_2^{\nu}]^{1/\nu} \tag{1}$$

where N_1 and N_2 are the aggregate amount of efficiency units of labor in occupation 1 and 2, respectively, $0 < \theta < 1$ governs the share of each occupation on total output and ν the elasticity of substitution between the two occupations.

Consumption of that final good is financed using labor earnings, as workers do not save and are born with zero wealth. Workers' preferences are described by a utility function of the constant relative risk aversion class. More specifically, given an amount of consumption c an individual ranks consumption levels c according to $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, with $\gamma > 1$.

Workers are endowed with a vector of occupation-specific abilities. These abilities can be thought of skills that are useful in a given occupation (for example, mathematical thinking for an engineer or physical strength for a construction worker). Some abilities may be innate but others can be the result of previously accumulated human capital. Nonetheless, we do not specify the origin of those abilities and we treat them as being predetermined at the time of the occupational choice. Abilities can be correlated across occupations and as a result some workers may excel at several professions. In what follows, the vector of abilities is denoted by $\boldsymbol{X} = (X_1, X_2)$. The elements of $\boldsymbol{X}$ is a Gumbel copula of two Fréchet distributions:

$$F(x_1, x_2) = Pr(X_1 < x_1, X_2 < x_2) = \exp\left\{ -\left[\sum_{i=1}^{2} (T_i^{\alpha} x_i^{-\alpha})^{1/\rho} \right]^{\rho} \right\} \tag{2}$$

The parameter ρ controls the dependence across ability levels for a given worker.

[3] Focusing on two occupations - one relatively risky and one relatively safe - is done only for simplicity. The framework can be easily generalized to an arbitrary number J of occupations.

The parameter α drives the dispersion and it is common to all abilities. Given (2), the marginal distributions are standard univariate Fréchet with cdf

$$Pr(X_i < x_i) = \exp\left\{-\left(\frac{x_i}{T_i}\right)^{-\alpha}\right\} \tag{3}$$

In section A of Appendix A we provide a derivation for this result. [4]

2.1 Occupational Choice and Sorting

Given a realization of $\boldsymbol{X} = (x_1, x_2)$ a worker may opt from one of two alternative careers. In one of them, earnings are more uncertain or risky and we assume, without loss of generality, that occupation 2 is the riskier one. The uncertainty is driven by shocks that alter a worker's ability to perform an occupation; shocks are distributed according to $F_i(y)$ for occupations $i = 1, 2$. We assume shocks are log-normal and have mean equal to one and $var(log(y_i)) = \sigma_i^2$. It is worth repeating—and this is what makes the problem interesting— that the occupational choice is conditional on the pre-determined abilities $\boldsymbol{X}$ but unconditional on the subsequent shock the worker experiences while on the job.

To formalize the occupational decision given $\boldsymbol{X}$ and the market prices for abilities in each occupation, w_1 and w_2, the value of working in occupation i is denoted by $V_i(x_i, w_i)$ and it is equal to:

$$V_i(x_i, w_i) = \max_c \int_{y \in \mathbb{Y}} \frac{c^{1-\gamma}}{1-\gamma}\, dF_i(y) \tag{4}$$
$$\text{subject to } c \leq x_i\, e^y\, w_i$$

To determine the value of working in an occupation the worker needs to know the price of a unit of ability in that occupation, denoted by w_i and the worker's own

[4] A similar approach is followed in Lind and Ramondo (2018).The authors augment a Ricardian trade model by using a multivariate max-stable Fréchet distributions to represent countries sectoral productivities.

pre-determined ability x_i. The price of the skills, w_i is determined in a competitive equilibrium but taken as given by the worker when deciding the occupation she is going to work for. Once on the job, the total amount of resources to consume are constrained by the total amount of ability $x_i e^y$ times its price w_i. As shocks y are stochastic with support $\mathbb{Y}$, the value of occupation i is given by the expected utility of consumption.

Among the two alternative careers, the worker picks the one with the highest value.

$$V(\mathbf{X}, w_1, w_2) = \max \{V_1(x_1, w_1), V_2(x_2, w_2)\} \tag{5}$$

Given that only two occupations are available, worker sorting in our environment is summarized by a share p_1 of workers choosing occupation 1.

Proposition 2.1 *The share of workers choosing occupation* 1*,*p_1*, is given by*

$$p_1 = \frac{T_1^{\frac{\alpha}{\rho}} |\Omega_1(w_1)|^{\frac{\alpha}{\rho(1-\gamma)}}}{\sum_{i=1}^{2} T_i^{\frac{\alpha}{\rho}} |\Omega_i(w_i)|^{\frac{\alpha}{\rho(1-\gamma)}}} \tag{6}$$

In section A.1 of Appendix A we provide a proof for this result. Note that the proportion of workers, everything else equal, increases with the increase in its wage rate as they feel attracted to that occupation. The same happens if T_1 is higher, everything else equal, the ability of workers is higher in occupation 1 relative to occupation 2 so they will have a comparative advantage for occupation 1 and thus more of them will choose that occupation.

Once we have found the probability that a worker chooses occupation 1, and therefore the mass of workers performing occupation 1, we need to derive the type of worker that picks that occupation. In other words, in order to calculate the total labor input in a given occupation it is necessary to know the average ability of its workers.

Proposition 2.2 *The amount of efficiency units in occupation i is*

$$N_i = p_i \mathbb{E}(\tilde{x}_i) = p_i^{\frac{\alpha-\rho}{\alpha}} T_i \Gamma \left(1 - \frac{1}{\alpha}\right)$$

where $\mathbb{E}(\tilde{x}_i)$ is the average ability of workers who choose occupation i (i.e. post-sorting).

In section A.2 of Appendix A we offer a proof for this proposition. It shows that the amount of labor in efficiency units in either occupation is only a function of the mass of workers and parameters related to the distribution of abilities. Note that $N_i = p_i^{\frac{-\rho}{\alpha}} p_i T_i \Gamma \left(1 - \frac{1}{\alpha}\right) = p_i^{\frac{-\rho}{\alpha}} \mathbb{E}(x_i)$ where $\mathbb{E}(x_i)$ is the average ex-ante ability (i.e. pre-sorting). Given that $\alpha > 2$ and $0 < \rho < 1$ then it is easy to see that average skills of the workers after sorting is higher than their ex-ante average skills. This is the direct consequence of sorting given the workers exhibit comparative advantages to work in occupation 1. When $\rho = 0$, that means, when there is perfect dependence of abilities, there is no sorting on relative skills or comparative advantages. In this special case workers are equally good (or bad) to work in either occupation so the sorting mechanism present in the model does not affect the average skill of workers once they have chosen an occupation.

2.2 The Competitive Equilibrium Allocation

A competitive equilibrium is a pair of employment levels (mass of efficiency units) N_1 and N_2 and a pair of wages w_1 and w_2, and an associated level of output Y_{CE}. The employment levels result from the solution to workers' occupational choice problem, and wages are such that the labor market for each occupation clears. Since labor markets are perfectly competitive the wage rate in a given occupation equals the marginal product of employment of that occupation.

Proposition 2.3 *The competitive equilibrium level of output Y_{CE} is given by*

$$Y_{CE} = \Bigg\{ \theta T_1^{\nu} \left[1 + \left(\frac{T_2}{T_1}\right)^{\frac{\alpha\nu(\rho-\alpha)}{(\nu(\rho-\alpha)+\alpha}) (\rho-\alpha)} \left(\frac{1-\theta}{\theta}\right)^{\frac{\alpha}{\nu(\rho-\alpha)+\alpha}} \left(\frac{E_2}{E_1}\right)^{\frac{\alpha}{(\nu(\rho-\alpha)+\alpha)(1-\gamma)}} \right]^{\frac{\nu(\rho-\alpha)}{\alpha}} +$$

$$(1-\theta) T_2^{\nu} \left[1 + \left(\frac{T_1}{T_2}\right)^{\frac{-\alpha\nu(\rho-\alpha)}{(\nu(\rho-\alpha)+\alpha}) (\rho-\alpha)} \left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha}{\nu(\rho-\alpha)+\alpha}} \left(\frac{E_1}{E_2}\right)^{\frac{\alpha}{(\nu(\rho-\alpha)+\alpha)(1-\gamma)}} \right]^{\frac{\nu(\rho-\alpha)}{\alpha}} \Bigg\}^{1/\nu}$$

$$\Gamma\left(1 - \frac{1}{\alpha}\right)$$

where $E_i = \mathbb{E}(e^{y_i(1-\gamma)}) = e^{(1-\gamma)(-\frac{\sigma^2\gamma}{2})}$

We provide a detailed derivation of this result in section A.3 of Appendix A. As it clears from the expression, the level of output depends on the technological requirements for each type of occupation $\left(\frac{1-\theta}{\theta}\right)$, the ratio $\left(\frac{T_2}{T_1}\right)$ that govern the gap in means of ex-ante abilities; and the ratio of earnings $\left(\frac{E_2}{E_1}\right)$. To illustrate some of the mechanisms at place it is worth analyzing the special case of Cobb-Douglas technology.

$$Y_{CE} = T_1^{\theta} \left[\frac{\theta E_1^{\frac{1}{1-\gamma}}}{\theta E_1^{\frac{1}{1-\gamma}} + (1-\theta) E_2^{\frac{1}{1-\gamma}}} \right]^{\frac{\theta(\alpha-\rho)}{\alpha}} T_2^{1-\theta} \left[\frac{(1-\theta) E_2^{\frac{1}{1-\gamma}}}{\theta E_1^{\frac{1}{1-\gamma}} + (1-\theta) E_2^{\frac{1}{1-\gamma}}} \right]^{\frac{(1-\theta)(\alpha-\rho)}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right)$$

Given the technological requirements, the level of output depends on the mass of efficiency units in each occupation. Everything else constant, the higher the earnings in an occupation, the higher the proportion of workers they will choose that occupation. As for the parameters T_i's that govern the relative mean of ex-ante abilities between occupations, as explained before it affects the proportion of workers that choose an occupation and, as clear in the expression of output, the mass of efficiency units in equilibrium. If workers were risk neutral, that is if γ were zero, the mass of workers would only depend on the expected value of earnings in a given occupation.

Note that E_i increases with γ or that $E_i^{1/(1-\gamma)}$ declines with γ, and the decline is larger the larger the variance of the idiosyncratic shock distribution. This result shows that relatively riskier occupations attract fewer workers in equilibrium, even though these may have high expected earnings.

3 The Compensation for Risk in the Labor Market

We use our framework to study the implications of imperfect risk-sharing for the risk-return trade-off workers face in the labor market. In other words, we aim to obtain expressions for the compensation for the permanent risk of labor earnings. We first focus on the relative price of the two types of human capital or the ratio of wage rates and then on the ratio of earnings.

3.1 The Wage Premium and the Compensation for Risk

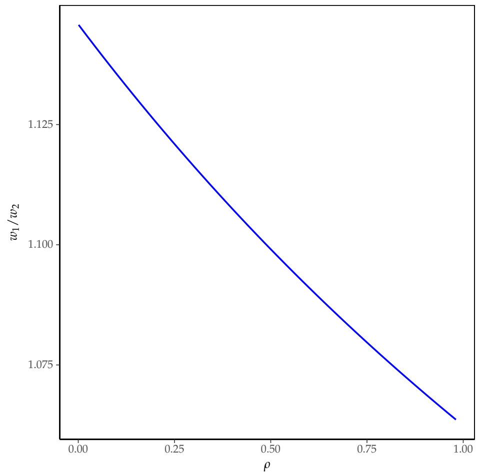
We aim to compute the relative price of the two types of human capital. In equilibrium the ratio of wage rates or prices is the ratio of marginal productivities. Using 1 and can be written as,

$$WP = \frac{w_1}{w_2} = \frac{\theta}{1-\theta}\left(\frac{N_1}{N_2}\right)^{\nu-1}. \tag{7}$$

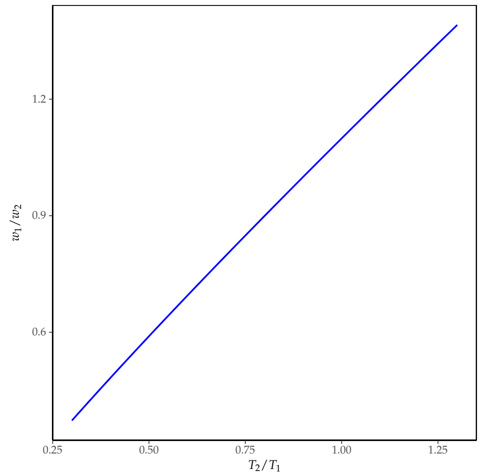
Using (33) to substitute for N_1/N_2 we have that

$$WP = \frac{w_1}{w_2} = \left(\frac{1-\theta}{\theta}\right)^{-\frac{\rho}{\nu(\rho-\alpha)+\alpha}} \left(\frac{E_2}{E_1}\right)^{\frac{(\alpha-\rho)(1-\nu)}{(\nu(\rho-\alpha)+\alpha)(1-\gamma)}} \left(\frac{T_1}{T_2}\right)^{\frac{\alpha(\nu-1)}{\nu(\rho-\alpha)+\alpha}} \tag{8}$$

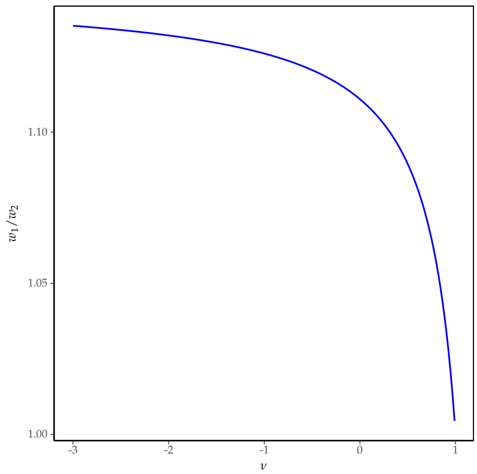
The ratio of wages has three components. The first term of the right hand side is related to the shape of the aggregate technology. Everything else constant, wages rise in occupation 1 if θ falls. The second term, represents the compensation for risk. This premium rises with γ and equals zero when $\gamma = 0$. It also rises with the spread between the variances of the idiosyncratic shocks. The third term relates the skills with the wage premium.



(a) Wage Premium and ρ.



(b) Wage Premium and T_1/T_2.



(c) Wage Premium and ν.

Figure 1: The three figures show how wage premium of the risky relative to the safe industry, varies for different values of three parameters: (a) ρ, (b) T_1/T_2, and (c) ν.

We are interested in studying how changes in the parameters of interest affect the relative price of the two types of human capital. This is shown in Figure 1. We begin by analyzing the changes in the ratio of wage rates w_1/w_2 for different values of ρ. This parameter governs the degree of dependence between the abilities of workers for working in one occupation or the other, also interpreted as the degree of comparative advantage. When ρ approaches to zero (to one) it means that the ability draws of a worker are very dependent (non-dependent) or, in other words, if a worker is good at performing one occupation there is also a high (low) probability of being also good at the other occupation. We can think of ρ approaching to zero as the limiting case in which there is only one ability to perform both occupations or, just absolute advantage. As it is clear in the picture, the higher the ρ the lower the relative wage rate in occupation 1. The reason in this case is very simple, when ρ is high then there is more selection in equilibrium. It is always the case that less workers will choose the risky occupation (because they are risk averse), but the higher the ρ the more selected they will be and thus with higher mean ability conditional on choosing occupation 1 (and thus efficiency units). Since the technology exhibits decreasing returns at the occupational level then the lower the relative wage.

In the second picture we plot the ratio of wages when the ratio T_2/T_1 changes. As T_2/T_1 increases, the abilities of occupation 1 (the risky) are relatively scarce and thus, everything else equal, one unit of human capital of occupation 1 is relatively more expensive. There will be more workers in occupation 1 but on average they will have less ability levels and that makes the relative wage to be higher.

The third picture shows the ratio of wage rates for different values of ν, starting at values close to zero (the Cobb-Douglas case). The figure shows that the more substitutable the occupations are when producing output, the lower the price of one unit of human capital in occupation 1 relative to occupation 2. When markets are incomplete risk averse workers try to avoid the risky occupation, because of the decreasing returns in each occupation the wage rate is higher and that is the way that the market attracts and compensates them. However, the more substitutable the two occupations are the lesser the wage rate will respond. This is why the higher the value of ν the lower w_1/w_2.

3.2 The Earnings Premium

Another object of interest is the earnings premium which is what is observed in the data. The earnings premium is defined as the ratio of average earnings in occupation 1 relative to occupation 2:

$$EP = \frac{\frac{w_1 N_1}{p_1}}{\frac{w_2 N_2}{p_2}} \tag{9}$$

In the Appendix we show that the earnings premium is equal to,

$$EP = \left(\frac{E_1}{E_2}\right)^{\frac{1}{\gamma-1}}. \tag{10}$$

Interestingly, the earnings premium only depends on the parameters that govern the risk premium, i.e. the relative variance of earnings shocks and the coefficient of risk aversion. As expected, the higher the value of γ the higher the ratio of earnings. This is clearly depicted in 2. Everything else equal the more risk averse a worker is the higher the compensation she/he requires to choose the risky occupation 1. For a fix value of the risk aversion parameter, the higher the volatility of shocks of occupation 1 relative to 2, the higher the compensation for the risk workers face.

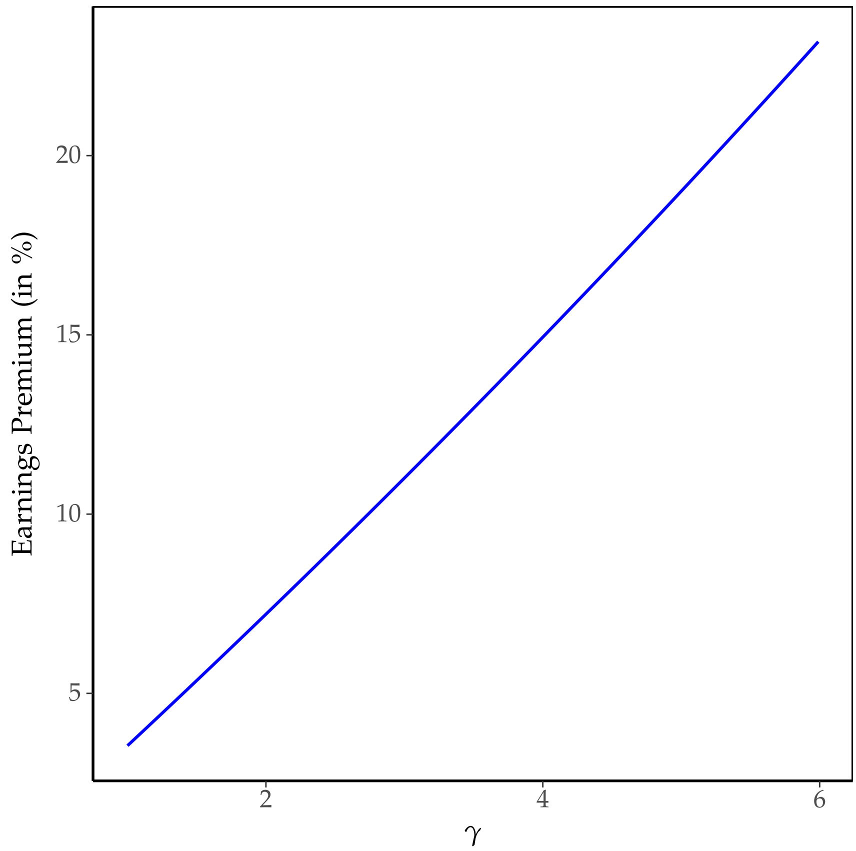


Figure 2: Risk Aversion and the Earnings Premium

Notes: The figure shows how the earnings premium defined as the average earnings of the risky occupation relative to earnings of the safe occupation varies with the risk aversion coefficient.

4 The Misallocation of Human Capital

Market incompleteness makes risky occupations less attractive. Consequently, a competitive equilibrium misallocates labor relative to an efficient allocation in which workers are sorted in order to maximize output. We begin by solving for the efficient level of output. We then compare its value to the allocation chosen by a social planner. We finally relate the degree of misallocation — the difference in output between the two allocations — to the parameters of interest.

4.1 The Social Planner's Problem

The efficient allocation is the one that a social planner decides if the objective is to maximize output and redistribute it across workers. The planner allocates workers across the two occupations after observing each worker's ability. Of course, the planner does not observe the shocks that workers receive once they work in an occupation. Therefore the planner makes the decision of where to allocate workers knowing only the ex-ante abilities (skills). Proposition 2.2 established the relationship between efficiency units in occupation i, N_i and its mass of workers, p_i.

Thus, we use it to solve the social planner's problem. The problem reduces to find the masses of workers in occupations 1 and 2, p_1^{SP} and p_2^{SP} that maximizes output.

$$\max_{p_1^{SP}, p_2^{SP}} \left[\theta T_1^{\nu} \left(p_1^{SP} \right)^{\nu \frac{\alpha-\rho}{\alpha}} + (1-\theta) T_2^{\nu} \left(p_2^{SP} \right)^{\nu \frac{\alpha-\rho}{\alpha}} \right]^{1/\nu} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{11}$$

subject to,

$$p_1^{SP} + p_2^{SP} = 1 \tag{12}$$

In the Appendix we solve for the social planner's problem and show that efficient output is given by:

$$\begin{aligned} Y_{SP} = \Bigg[\theta T_1^{\nu} & \left(\frac{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}} \frac{T_2}{T_1}^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}}}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}} \frac{T_2}{T_1}^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}} + 1} \right)^{\nu \frac{\alpha-\rho}{\alpha}} + \\ (1-\theta) T_1^{\nu} & \left(\frac{1}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}} \frac{T_2}{T_1}^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}} + 1} \right)^{\nu \frac{\alpha-\rho}{\alpha}} \Bigg]^{1/\nu} \Gamma\left(1 - \frac{1}{\alpha}\right) \end{aligned} \tag{13}$$

If we again consider the special case of a Cobb-Douglas technology, then $p_1^{SP} = \theta$, $p_2^{SP} = 1 - \theta$. Therefore, the proportion of workers in each occupation is given by the technological requirement which is governed by θ. Equations (59) and (60) become

$$N_1^{SP} = T_1 \theta^{\frac{\alpha-\rho}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{14}$$

$$N_2^{SP} = T_2 (1-\theta)^{\frac{\alpha-\rho}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right). \tag{15}$$

Thus, the efficient level of output is

$$Y_{SP} = N_1^{SP\theta} N_2^{SP1-\theta} \Gamma\left(1 - \frac{1}{\alpha}\right) = T_1^{\theta} \theta^{\frac{\theta(\alpha-\rho)}{\alpha}} T_2^{(1-\theta)} (1-\theta)^{\frac{(1-\theta)(\alpha-\rho)}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{16}$$

As it is clear, by looking at the expression of Y_{CE}, when workers are risk neutral the level of output in competitive equilibrium is the same as the one obtained by the social planner. The risk consideration does not affect the allocation of resources, those are just given by the technology and the distributions of ex-ante abilities.

Figure 3 helps to clarify the intuition for why market incompleteness misallocates workers across occupations. Suppose a world with no ex-ante abilities but with post-entry uninsurable risk. The figure shows aggregate output as a function of employment in occupation 1 (the riskier occupation), fixing the value of N_2 for exposition purposes. In a competitive equilibrium workers are indifferent between the two occupations. Occupation 1 is riskier than 2 and therefore its wage must compensate workers for bearing a higher risk. For the wage to be high enough there have to be few workers in that occupation; there are diminishing marginal returns to either type of labor. This low level of employment corresponds to the value N_1^{CE} in the figure. The marginal product (the wage rate in equilibrium) is equal to the slope of the production function at that value. Thus, although risk is compensated in the competitive equilibrium what is interesting even in this simple problem is that this allocation does not maximize output. A social planner can increase output by reallocating workers across occupations. That is, the social planner makes risk irrelevant so at the first-best allocation the amount of employment in occupation 1 is higher (N_1^{SP}). The corresponding marginal product is lower as shown by the flatter slope. Because the amount of employment in the planner's problem is the one that maximizes output, the competitive equilibrium leads to a risky occupation that is too small. With ex-ante abilities in equilibrium not all workers are indifferent between the two occupations, but the intuition is the same.

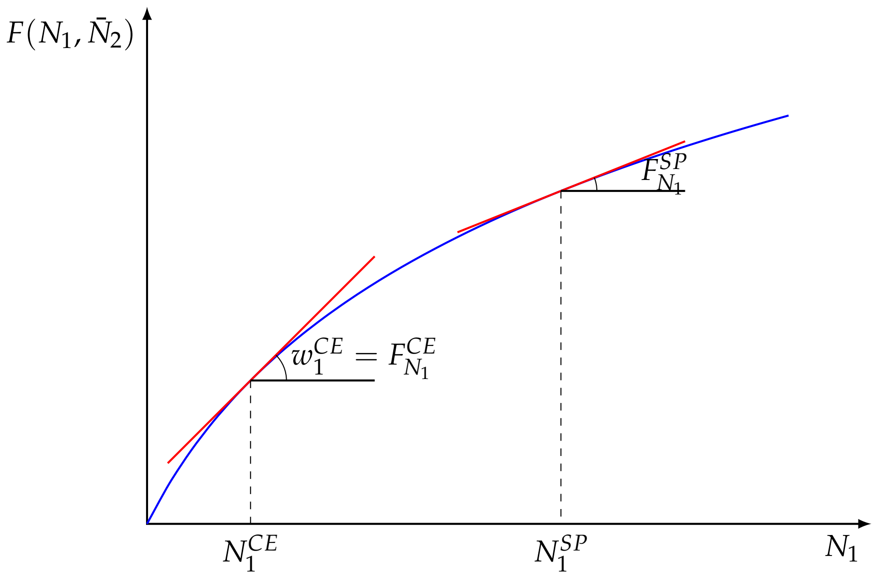


Figure 3: Risk, Compensating Differential and the Optimal Allocation

Notes: The figure shows the level of output and the allocation of workers in the risky industry (N_1) a simplified version of the model both in the laissez faire competitive equilibrium (N_1^{CE}) and in the social planner problem (N_1^{SP}).

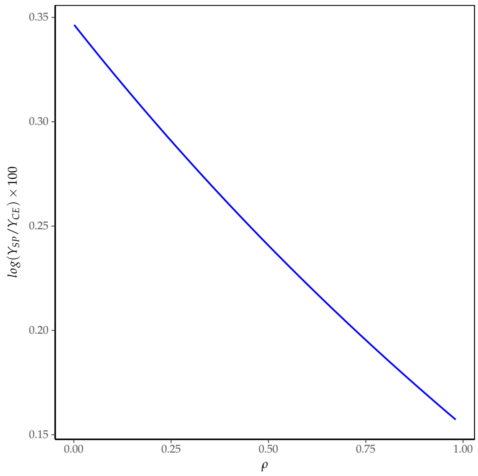
4.2 Risk, Abilities and The Degree of Misallocation

We are interested in studying the degree of misallocation implied by market incompleteness. In Figure 4 we plot the log of the ratio of Y_{SP}/Y_{CE} (in percentage terms) for different values of the parameters of interest.

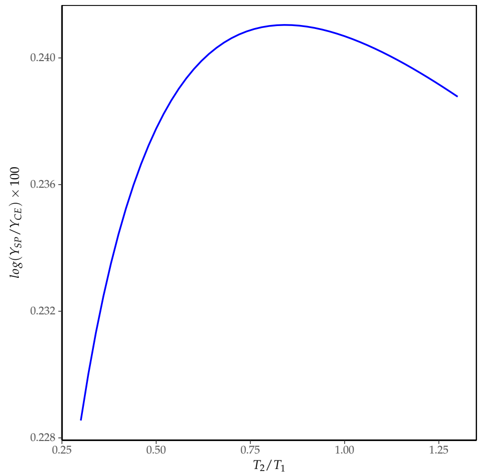
We begin by analyzing the degree of misallocation for different values of the parameter ρ. This parameter governs the degree of dependence between the abilities of workers for working in one occupation or the other, also interpreted as the degree of comparative advantage. When ρ approaches to zero (to one) it means that the ability draws of a worker are very dependent (non-dependent) or, in other words, if a worker is very good at performing one occupation there is also a high (low) probability of being also good at the other occupation. We can think of ρ approaching to zero as the limiting case in which there is only one ability to perform both occupations or, just absolute advantage. As it is clear in the picture, the higher the ρ the close is the competitive equilibrium allocation to the optimal allocation. The reason in this case is very simple, although less workers will choose the risky occupation compared to the social planner allocation, the higher the ρ the more selected they will be and thus with higher mean ability in equilibrium. Therefore, the degree of comparative advantages or selection alleviates the negative effect of market incompleteness on output.

We also plot the degree of misallocation for different values of the ratio of the mean of ex-ante abilities T_2/T_1. As the figure shows, for relatively low or high values of T_2/T_1 the competitive equilibrium allocation is closer to the optimal allocation. When T_2/T_1 is low, everything else equal, the abilities of occupation 1 (the risky) are relatively abundant so even though less workers will choose that occupation in the competitive equilibrium (compared to the social planner allocation) the mass of efficiency units will be larger and that gets the output of the economy closer to its optimal level. When T_2/T_1 is high, everything else equal, the abilities of occupation 1 (the risky) are relatively scarce. Therefore, occupation 2 is relative more important for the planner to maximize output and so the optimal quantity of workers is relatively higher in that occupation. Therefore, occupation 1 is not that important and thus the gap in the number of workers between the competitive equilibrium allocation and the social planner allocation is not that consequential for the level of output gap.

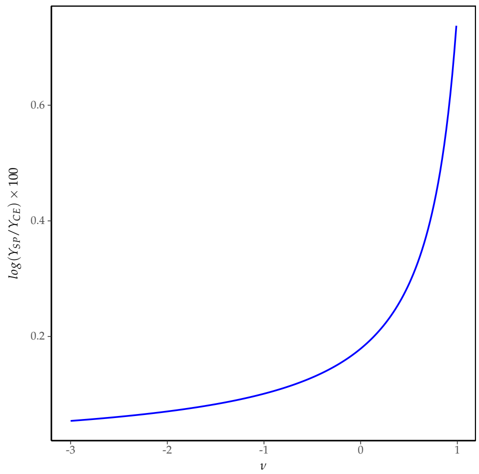
Lastly, the figure shows the how the degree of misallocation varies with ν, starting at values close to zero (the Cobb-Douglas case) and letting it grow to make the occupations more substitutable in output. The figure shows that the more substitutable the occupations the higher the distance between the competitive equilibrium and the optimal allocations. When markets are incomplete risk averse workers try to avoid



(a) Misallocation and ρ.



(b) Misallocation and $T1/T2$.



(c) Misallocation and ν.

Figure 4: The three figures show how the degree of misallocation varies for different values of three parameters: (a) ρ, (b) T_1/T_2, and (c) ν. Misallocation is measured by the percentage deviation of the competitive equilibrium output (Y_{CE}) from the first-best (Y_{SP}).

the risky occupation, because of the decreasing returns in each occupation the wage rate is higher and that is the way that the market attracts them. However, in equilibrium the proportion of workers is less than the one allocated by the planner. The wage rate raises but it is not enough to obtain the optimal level of workers. However, the more substitutes the occupations are, the less the wage rate will respond and thus less workers will choose the risky occupation in equilibrium. As a result, the farer the competitive equilibrium output will be from its optimal level.

4.3 Corrective Taxation

Proposition 4.1 *If wages in occupations 1 and 2 are taxed by occupation-specific taxes τ_1 and τ_2, respectively, the social planner's allocation is achieved by setting taxes such that*

$$\frac{1-\tau_1}{1-\tau_2} = \left(\frac{E_1}{E_2}\right)^{\frac{1}{\gamma-1}}$$

where $E_i = \mathbb{E}(e^{y_i(1-\gamma)})$. Furthermore, if taxes are chosen so that government's budget remains balanced, the tax rates are given by

$$\tau_1 = \frac{1-\left(\frac{E_2}{E_1}\right)^{\frac{1}{1-\gamma}}}{1+\left(\frac{T_2}{T_1}\right)^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}}\left(\frac{1-\theta}{\theta}\right)^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}}\left(\frac{E_2}{E_1}\right)^{\frac{1}{1-\gamma}}}$$

and

$$\tau_2 = -\left(\frac{T_2}{T_1}\right)^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}}\left(\frac{1-\theta}{\theta}\right)^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}}\tau_1$$

In section A.4 of Appendix A we prove this results. The most interesting aspect to note is that the ratio of tax rates given in the proposition, that is $\frac{1-\tau_1}{1-\tau_2}$, is the same as the one given in the earnings premium in equation (10). The reason is simple. The taxes try to correct the misallocation generated in competitive equilibrium by encouraging more workers to choose the risky occupation. The way to do that is to tax relatively more the workers that choose occupation 2, the safe occupation. As it clearly transpires from the expression, the higher the variance of income shocks of occupation 1 relative to occupation 2, everything else equal, the higher is the ratio is $\left(\frac{E_1}{E_2}\right)^{\frac{1}{\gamma-1}}$. Thus, the higher is the relative tax rate on earnings of the workers of the safe occupation (τ_2) relative to the workers in the risky occupation (τ_1). Similarly, for the same variance of the shocks, the higher the risk aversion parameter, the higher the degree of misallocation and thus the higher the the tax rates on earnings of workers in the safe occupation relative to the workers in the risky occupation that is needed to obtain the social planner allocation.

5 Quantitative Analysis

We are interested in using our theory to measure the degree of misallocation due to market incompleteness. We take our model to the data and give quantitative answers to questions in labor economics, international trade, and development economics.

First, we study the misallocation of workers across industries in the US. As has been documented in the literature, industries vary by earnings risk and this risk is mostly uninsurable. Second, motivated by the growing literature in international trade linking trade reforms to increasing volatility in workers' earnings, we measure the change in welfare associated with the rapid increase in the import penetration experienced by the US manufacturing sector. Third, we measure the productivity gap in Ethiopia due to the misallocation of workers across rural and urban areas.

5.1 Labor Income Risk and the Misallocation of Workers Across US Industries

We use the results of Cubas and Silos (2017) and several other moments from US earnings data, to calibrate the model. Using the Survey of Income and Program Participation (SIPP) as the source of earnings data, Cubas and Silos (2017) decompose individual-level earnings in each US industry into a permanent and transitory component. They estimate the variance of each component, reporting results for a total of 19 industries.

According to the estimates reported, industries vary greatly in the degree of permanent earnings volatility. We use their estimates and divide industries in two groups, the "risky" and the "safe" sector, according to the variance of the permanent component of earnings.[5]. The first group has a permanent variance of 0.0057 and the second a variance of 0.00399. Cubas and Silos (2017) estimate a random walk process for the permanent component of earnings. Because our model is static, we assume a 40-year career for workers and thus multiply by 40 the value of each variance. In other words, this product represents the variance of the permanent component to earnings over a worker's life-cycle .

We need to calibrate the parameters of the copula, T_1, T_2, α and ρ, in addition to the aggregate technology parameters θ and ν, and the risk aversion parameter γ. Because in our general equilibrium framework mean earnings does not depend on the scale parameters of the Frechet distribution (T_1 and T_2) we fix them at a value of one. To calibrate α we employ the following procedure. Using the 2001 panel of the SIPP we estimate a fixed-effects regression for individual earnings controlling by age and time (the SIPP is a quarterly panel). We interpret the distribution of fixed effects as the distribution of workers' productivities prior to experiencing shocks. Consistent with this interpretation we use the standard deviation of fixed effects across workers to calibrate a value of α. Because α is the same for the two abilities distributions, we target the standard deviation of (log) abilities of the safe industry. The standard deviation of workers' fixed effects in the safe industry is 0.345 in the data. We estimate the share parameter θ in the aggregate technology by setting it so that the model delivers a share of workers in the risky industry of 75%, the value in data. Finally, to estimate the risk aversion coefficients we can use (10). That equation states that the ratio of average earnings across the two industries depends only on the risk aversion parameter γ and the two standard deviations of earnings shocks. The earnings premium across the two industries is 6.75%, yielding a risk aversion coefficient of 2.92.

[5]The "risky" group includes Utilities, Finance, Non Durable Goods Manuf., Wholesale Trade, Communication, Retail Trade, Medical Services, Transportation, Recreation and Entertainment, Construction, Durable Goods Manuf. and Other Services. The "safe" group includes Agriculture and Forestry, Social Services, Government, Hospitals, Business Services, and Personal Services.

Because we use the standard deviation of earnings to estimate α and the share of workers in the risky industry to estimate θ, we can't separately estimate ρ. We opt to analyze the model by assuming a range of values of ρ (the minimum is 0.1 and the maximum is 1), recalibrating θ and α for each value of the dependency parameter.[6] Lastly, the parameter ν drives the elasticity of substitution across occupations. The literature lacks a clear reference for an estimate of this elasticity. We opt for a value of ν equal to 1/3 (an elasticity of 1.5). The implied elasticity of that value is halfway between the Cobb-Douglas case (ν equal to 0 or a unit-elasticity of substitution) and an elasticity of substitution equal to 3 (or ν equal to 2/3) used by Hsieh and Klenow (2009).

Figure 5 shows the distance between the value of output in the competitive equilibrium and the value obtained in the social planner's problem for different values of ρ and γ.

As in Figure 4, as ρ increases the degree of misallocation decreases. The logic and intuition is the same: independent draws imply a higher degree of selection because good abilities can only be used in one occupation. When the dependence is low for both abilities; with low dependence there is a higher likelihood that the worker has high ability in at least one occupation. The higher selection – i.e. the higher ability by occupation – implies a better buffer against risk and therefore the absence of insurance markets matters less. In addition, for a fixed ρ, the higher the value of the risk aversion parameter γ, the higher the degree of misallocation. As risk aversion rises, entering the risky industry is less desirable. Higher risk aversion exacerbates the costs of market incompleteness. These results provide a quantitatively plausible range of the level of misallocation. The minimum loss is 0.1% and the maximum loss is around 0.6% of output, permanently.

[6] This procedure delivers a range of values for θ between 0.698 and 0.716.

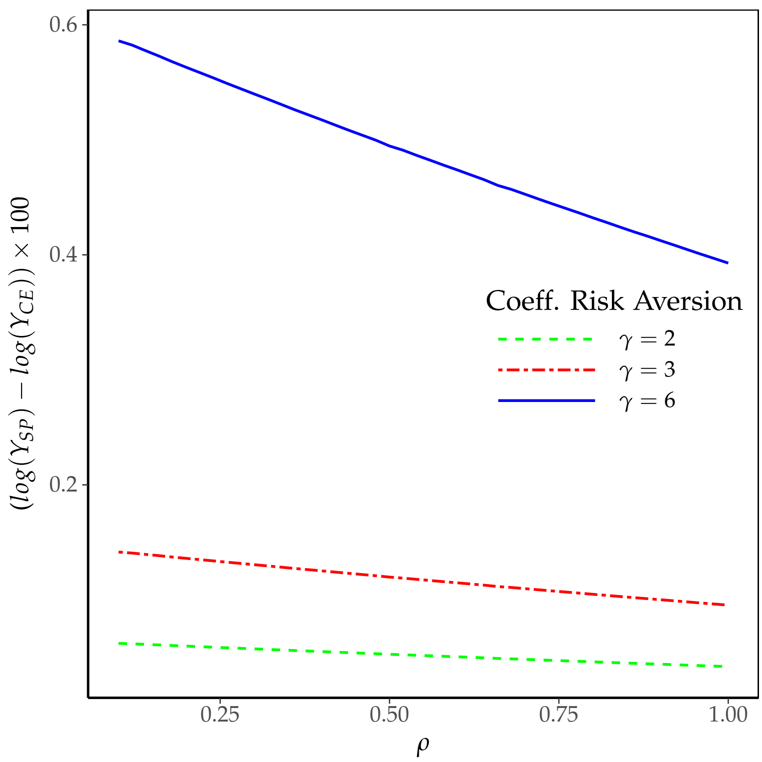


Figure 5: The Degree of Misallocation Across Industries

Notes: The figure plots the degree of misallocation. The degree of misallocation is measured as the percentage deviation of output in a competitive equilibrium from output at the social optimum; i.e. by the percentage deviation of the competitive equilibrium output (Y_{CE}) from the first-best (Y_{SP}). The horizontal axis represents different values for ρ. The three different lines represent different levels of risk aversion γ.

5.2 Risk, Import Penetration and the Misallocation of Workers

As has been documented by a large body of literature on labor and trade, the increase in import competition has dramatically changed US labor markets. According to Acemoglu, Autor, Dorn, Hanson, and Price (2016) the import penetration in the US manufacturing sector has increased by around 7 percentage points in the period 1991-2009. An important aspect related to this change is the increased importance of China as a competitive producer of manufactures after it entered the World Trade Organization. These authors document that the increase in import penetration of manufactures in the US accounts for a total loss of 12% of manufacturing employment in the United States.[7]

Interestingly, there is a growing number of studies that relate the degree of import penetration and trade exposure of an industry with the volatility of workers' earnings in that industry. Trade openness reallocates workers across sectors and across firms within sectors. Workers are heterogeneous and thus differently affected by the resulting reallocation. An important paper in this literature is Krishna and Senses (2014) who document that a 10% increase in import penetration in an industry is associated with a 23% increase in the variance of permanent shocks to labor earnings.

We use our framework to connect these two strands of the literature. We examine the output costs derived from the increase in import penetration in the tradable sector that results in worker reallocation. This reallocation occurs because the increase in risk due to trade openness makes the tradable sector less attractive. We use our previous calibration but we now divide industries in two groups: "tradables" and "non-tradables". The tradable group comprises Durable Goods Manufacturing, Non-Durable Goods Manufacturing and Agricultural and Forestry. The rest of the industries are included in non-tradables. The variance of the permanent shocks to earnings is 0.0061 and 0.0050 for the tradable sector and non-tradable sector, respectively. We interpret the allocations of our model with this parameterization as an initial steady state and entertain a trade reform to measure the change in the degree of misallocation. For that purpose, we use the estimates of Acemoglu, Autor, Dorn, Hanson, and Price (2016) who document an increase in the import penetration in the manufacturing sector of 7%. In addition, according to estimates of Krishna and Senses (2014), an increase of import penetration of 7%, corresponds to an increase in the variance of the permanent shock to labor earnings of the tradable sector of 16.1%. Thus, according to our estimates, the variance of the tradable sector would be 0.007. Ceteris paribus, in the new equilibrium with a riskier tradable sector the model

[7] Acemoglu, Autor, Dorn, Hanson, and Price (2016) report employment losses of about 2.2 million. Manufacturing employment in January of 1999 was about 17 million workers.

predicts an increase in the degree of misallocation and a decrease in the number of workers in the tradable sector.

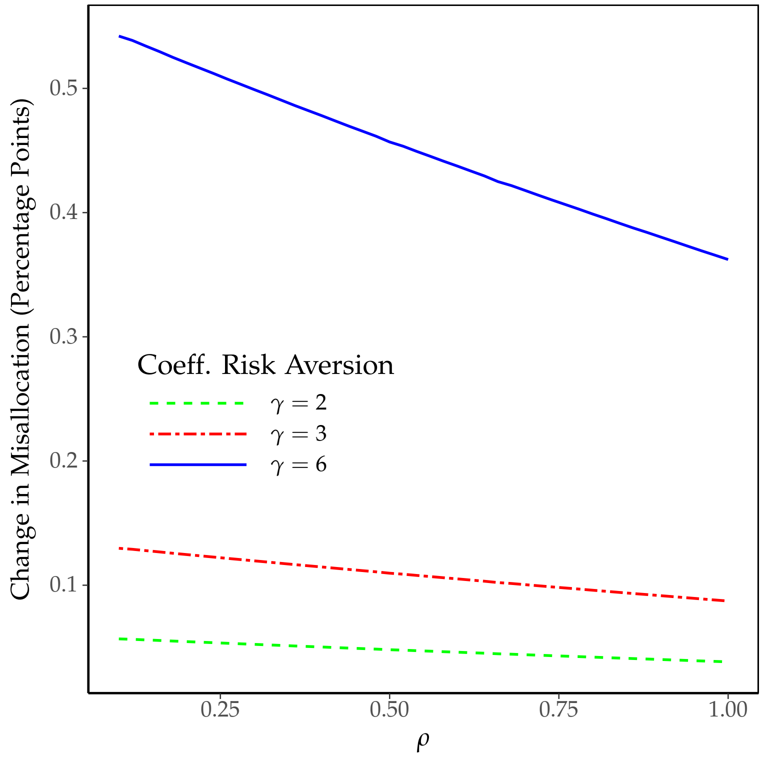


Figure 6: Import Penetration and Misallocation

Notes: The figure shows the change in the degree of misallocation for different values of ρ and γ. The degree of misallocation is measured as the percentage deviation of output in a competitive equilibrium from output at the social optimum; i.e. by the percentage deviation of the competitive equilibrium output (Y_{CE}) from the first-best (Y_{SP}). The horizontal axis represents different values for ρ. The three different lines represent different levels of risk aversion γ.

Figure 6 shows the change in the degree of misallocation for different values of ρ and γ. We measure misallocation the same way as before: the percentage change of the competitive equilibrium output from the first best. The figure plots the change in misallocation as trade opens. For example, if misallocation is 1% pre-trade and 1.5% post-trade, the change in misallocation is half a percentage point. As is clear from the graph, for a given value of ρ and γ, there is an increase in the degree of misallocation as a result of the trade reform. After the increase in trade openness the tradable industry is relatively more risky than the non-tradable industry. As a result,

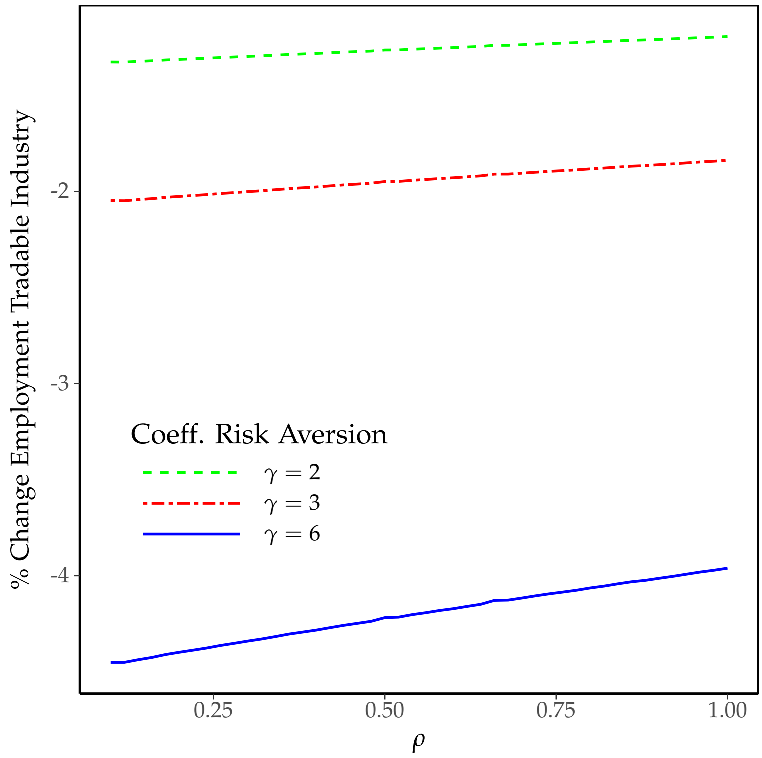


Figure 7: Change in the Size of the Risky Industry

Notes: The figure shows the change in the size of the risky industry in the competitive equilibrium allocation for different values of ρ and γ. The horizontal axis represents different values for ρ. The three different lines represent different levels of risk aversion γ.

workers leave the tradable sector, resulting in an allocation that is ever farther away from the first best than the pre-trade allocation was. The magnitude of this increase in misallocation depends upon the values of ρ and γ. As the picture shows, the increase in misallocation can plausibly be as large as 0.7 percentage points. Changes of this magnitude require abilities to be highly dependent and workers to be quite risk-averse.

Acemoglu, Autor, Dorn, Hanson, and Price (2016) estimate the employment losses in manufacturing due to import competition to be about 12% between 1999 and 2011. Our model predicts also a shrinking tradable sector as the earnings volatility rises relative to the non-tradable sector. Figure 7 shows employment losses in the tradable sector (as a percentage loss For low ρ and high γ, the employment losses are about

4.5%. For empirically plausible values of γ the employment losses are only slightly above 2%.

In response to a well identified shock such as an increase in trade openness, a government may want to correct the extra inefficiency through a corrective tax. In Section 4.3 we derive the ratio of earnings taxes in both occupations that delivers the social optimum. With the calibration we use to examine the losses from import penetration, we calculate the taxes needed to eliminate the inefficiency arising from an increase in trade openness.

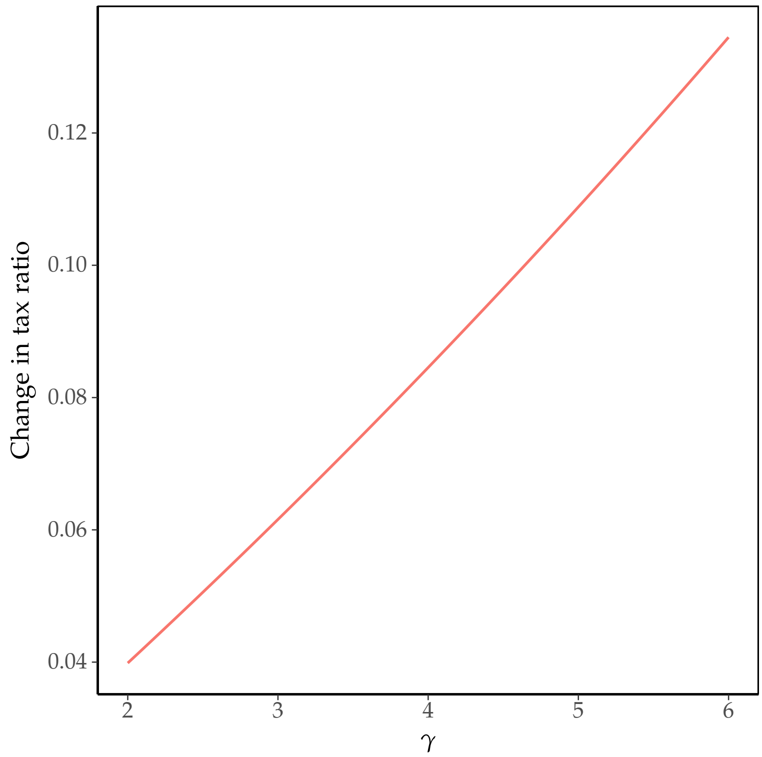


Figure 8: Import Penetration and Corrective Taxes

Notes: This figure shows the ratio in the level of taxes of the safe occupation relative to the risky occupation ($\frac{1-\tau_1}{1-\tau_2}$) that is needed to achieve the social planner allocation after the import penetration shock. The ratio is shown for different values of the risk aversion parameter γ.

Figure 8 plots the change in the ratio $\frac{1-\tau_1}{1-\tau_2}$ before and after the import penetration shock. The average ratio — across the range of the values of γ considered —is about 1.08 rising to 1.16 after the import penetration shock. An increasing ratio implies that the denominator (numerator) declines (rises), which implies a relative subsidy of the risky industry. Normalizing τ_1 to be 1 these results imply that the tax rate on the safe industry roughly doubles (from 7.4% to 13.8%).

5.3 Risk and the Misallocation of Rural and Urban Workers: The Case of Ethiopia

Differences in income per capita across countries are largely accounted for by differences in total factor productivity (TFP). As has been studied in many important papers (see e.g. Hsieh and Klenow (2009), Restuccia and Rogerson (2013), Lagakos and Waugh (2013), Lagakos, Mobarak, and Waugh (2018), Vollrath (2009), Midrigan and Xu (2014), Guner, Ventura, and Yi (2008)) misallocation of factors of production across firms, sectors or regions within an economy are important to explain cross-country TFP differences. In particular, a strand of literature has focused on the differences in labor productivity between urban and rural areas in poor countries (see e.g. Restuccia, Yang, and Zhu (2008), Adamopoulos, Brandt, Leight, and Restuccia (2017), Gollin, Lagakos, and Waugh (2014) and Herrendorf and Schoellman (2015)). The results of this research are relevant for understand the development problem because labor productivity is lower in rural areas but at the same time these areas concentrate much of the labor force.

Our framework is particularly related to the strand of this literature that highlights the importance of the lack of insurance markets in explaining the lack of migration to urban areas and thus the low observed productivity levels (see e.g. Harris and Todaro (1970), Townsend (1994), Udry (1994), Munshi and Rosenzweig (2016), Donovan (2020) and Morten (2019)). The idea is that although income is more volatile in rural areas, individuals living in those areas rely on informal insurance arrangements that are absent if they migrate to the city. For this reason they remain in rural areas. Although there is no consensus in the literature (see Lagakos for an excellent review), our model can contribute to the discussion on the importance of market incompleteness in explaining the observed low productivity levels and the larger share of rural workers in poor countries.

Our model highlights the interaction between skills and risk in shaping a worker's occupational or regional choice. In light of some of the results of this literature, the existence of informal insurance markets in the rural areas, everything else constant, makes the rural occupations more attractive to workers. Therefore, from the perspective of a social planner, a larger than desirable proportion of workers choose rural occupations. In addition, the more able workers are the ones choosing an urban occupation.

We use data from Ethiopia to calibrate our model and to measure the degree of misallocation of workers between rural and urban areas. We use the last two waves of the Ethiopia Socieconomic Survey (ESS) conducted in 2013/2014 (ESS2) and 2015/2016 (ESS3) obtained from the World Bank. These surveys are nationally representative and, the ESS2 and ESS3 together represent a panel of households and individuals for rural and all urban areas. We focus on two groups, urban and rural workers.

Of the total sample we consider 19,917 of individuals that we could match on both surveys. Our model points to the differential earnings risk as the reason behind the large observed proportion of rural workers. As documented in an extensive literature, although they have better opportunities in urban areas, rural workers have access to informal insurance mechanisms. Workers are risk averse and in light of our model, higher earnings risk in urban areas results in a portion of workers with a comparative advantage to work in urban areas remaining in rural communities. Consequently the rural sector is larger than it would be were the urban areas to have similar insurance opportunities.

To quantify the effect of this channel in explaining the large rural sector observed in Ethiopia we first need to estimate a measure of the volatility of earnings for both types of workers. Ideally one would use a long panel of individuals with the longest possible labor market histories. For the case of Ethiopia we only have two observations for individuals so we use them to compute the weighted variance of income growth between both periods.[8] Table 1 provides descriptive statistics of the variables to be included in the analysis for both types of workers. In the first row we present the percentage of rural and urban workers in our 2013 sample. As it is well known, this poor country exhibits a large proportion of the population in the agricultural sector. We also show the weighted average of labor earnings for both type of workers. As expected the urban wages are substantially higher than rural wages (25% higher). The third column presents our measure of volatility of earnings for both types of workers, it is 11% higher for urban workers. We also report the standard deviation of log earnings for the cross-section of workers in 2013. As expected it is higher for urban workers, 0.620 versus 0.591.

Table 1: Ethiopia Socioeconomic Survey (ESS):
Summary Statistics

Percent Rural	72%
Average Earnings Rural	1,268
Average Earnings Urban	1,582
Std. Dev. (Log) Earnings - Rural	0.62
Std. Dev. (Log) Earnings - Urban	0.59
Std. Dev. Earnings Growth - Rural	0.42
Std. Dev. Earnings Growth - Urban	0.47

Note: This table presents the summary statistics of the variables we use for our quantitative exercise for Ethiopia obtained from the Ethiopia Socieconomic Survey (ESS) conducted in 2013/2014 (ESS2) and 2015/2016 (ESS3).

We follow a similar parameterization as that for the US. We use moments of the cross-sectional earnings and the fraction of workers in the urban sector to pin down α and θ. However, we use the same value for the coefficient of risk aversion as that

[8] The sample size is substantially reduced when we strict our analysis to positive earnings matching individuals present in both waves.

calibrated to US data (2.9). The reason is that the earnings differential in Ethiopia between the urban and rural is large (about 25%). This differential yields a risk aversion coefficient that is unreasonably large (slightly above 8). Such a value amplifies the costs of misallocation. As previously discussed we are not able to identify the parameter ρ so we find values of α and θ for a sequence of values of ρ, $(0.1, \ldots, 0.99)$. For each value of ρ we choose α and θ to match the standard deviation of log earnings in 2013 and the percentage of urban workers. This gives us a range of values for θ and α (one per value of ρ). There is minimal variation in the calibrated values for α with an average of 3.11. As for θ it is in the range of 0.02 and 0.34 with an average of 0.2. With these values for the parameters we provide our estimates for the degree of misallocation for different values of ρ and risk aversion. This is shown in Figure 9.

The range of values for misallocation can be as high as a fourth of a percentage point of output. However, a fairly high risk aversion coefficient 6 and a low value of ρ are necessary.

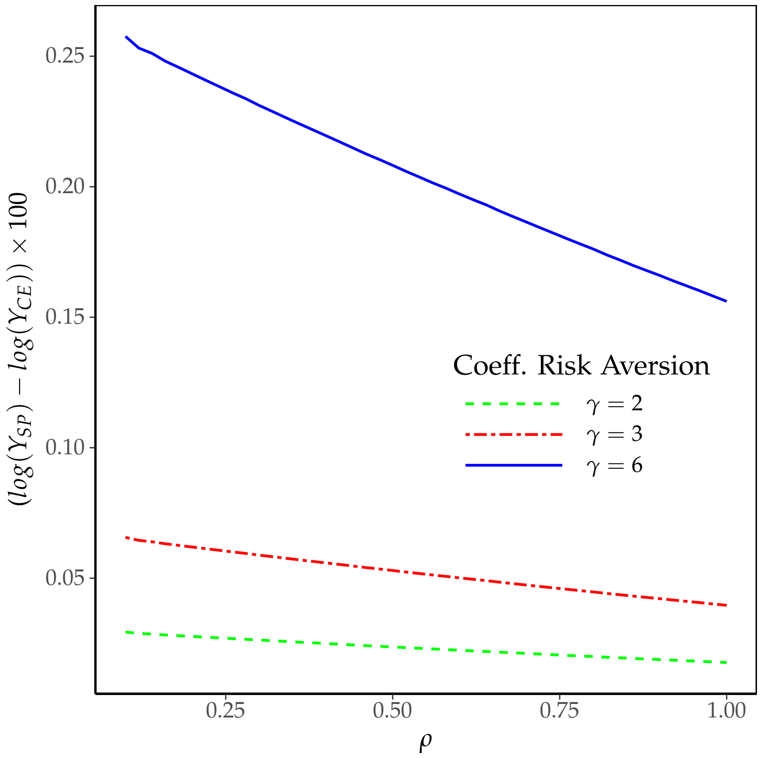


Figure 9: Misallocation of Urban and Rural Workers in Ethiopia

Notes: The figure shows the change in the degree of misallocation for different values of ρ and γ. The degree of misallocation is measured as the percentage deviation of output in a competitive equilibrium from output at the social optimum; i.e. by the percentage deviation of the competitive equilibrium output (Y_{CE}) from the first-best (Y_{SP}). The horizontal axis represents different values for ρ. The three different lines represent different levels of risk aversion γ.

6 Conclusions

How does the lack of insurance markets to insure against worker's permanent earnings shocks affect their occupational choice and the allocation of human capital in an economy? What are the consequences for aggregate productivity? We have answered these questions by developing a Roy model of occupational choice. Risk averse workers choose an occupation based on the occupation-specific risk they face and on their comparative and absolute advantages. The tractability of the Frechet distribution allows for a closed-form solution of the competitive equilibrium allocation. We obtain analytical expressions for the compensation for risk in the labor market, as well as for the aggregate level of human capital and output. In a competitive equilibrium, human capital is misallocated because workers avoid risky industries. The social planner allocates more workers to risky industries. The higher the risk aversion and the lower the degree of comparative advantage, the larger the misallocation. A relatively high ability in an occupation acts as a buffer against risk. Taxing safe occupations and subsidizing risky occupations restores the first-best in a competitive equilibrium

Several quantitative exercises estimate the size of misallocation due to market incompleteness. In our first exercise, using estimates from the literature of industry-level permanent shocks to earnings, we estimate a permanent output loss of 0.6% due exclusively to market incompleteness. Trade reforms during the 1990s increased the import penetration in the US manufacturing sector. Exposed industries saw a rise in earnings volatility. This rise in (uninsurable) risk reduced the size of the manufacturing sector; the model accounts for about a third of the observed drop in employment. The resulting inefficient allocation of human capital caused a permanent drop in output of 0.7 percentage points. Finally, our third exercise examines the rural-urban gap in developing countries. Using micro data from Ethiopia we find that workers in the urban sector earn more, but are also more exposed to consumption risk. The higher risk in urban areas increases the size of the rural sector (relative to the first best). The result is a permanently lower aggregate productivity of the order of a quarter of a percentage point.

This paper offers a new perspective for understanding the link between risk in labor markets and the aggregate levels of human capital. We also provide new insights on how missing insurance markets affect aggregate productivity. To focus on our proposed main mechanism, we abstract from many aspects of the labor market and the career choice of the individuals. For instance, we take earnings volatility as exogenous and we do not consider heterogeneity in risk aversion. For the sake of tractability and to obtain analytical expressions we also abstract from the career dynamics and the role that savings play in shaping the occupational choice. We also abstract from many barriers that surely affect the occupational choice and mobility of workers and that interact with the lack of insurance. From this perspective, we think our measured misallocation can be a lower bound in our quantitative exercises. We hope our findings encourage future research that relaxes these assumptions and that allows a better identification of the correlation of abilities.

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Figures

A Appendix

Frechet Marginal Distributions

Given a joint cumulative distribution $F_{x_1,x_2}(x_1, x_2)$ with support $(0, \infty) \times (0, \infty)$, the marginal distribution of x_2 is given by

$$f_{x_2}(x_2) = \int_0^\infty f_{x_1,x_2}(x_1, x_2) dx_1 \tag{17}$$

where the joint density is obtained from

$$f_{x_1,x_2}(x_1, x_2) = \frac{d^2}{dx_1 dx_2} F_{x_1,x_2}(x_1, x_2)$$

For the Gumbel copula with Frechet distribution

$$F_{x_1,x_2}(x_1, x_2) = \exp\left(-\left(x_1^{-\frac{\alpha}{\rho}} + x_2^{-\frac{\alpha}{\rho}}\right)^\rho\right)$$

differentiating once with respect to x_2 gives an expression for the joint density:

$$f_{x_1,x_2}(x_1, x_2) = \frac{d}{dx_1} \exp\left(-\left(x_1^{-\frac{\alpha}{\rho}} + x_2^{-\frac{\alpha}{\rho}}\right)^\rho\right) \left(x_1^{-\frac{\alpha}{\rho}} + x_2^{-\frac{\alpha}{\rho}}\right)^{\rho-1} \alpha x_2^{-\frac{\alpha}{\rho}-1}$$

Using this in (1) gives

$$
\begin{aligned}
f_{x_2}(x_2) &= \int_0^\infty \frac{d}{dx_1} \exp\left(-\left(x_1^{-\frac{\alpha}{\rho}} + x_2^{-\frac{\alpha}{\rho}}\right)^\rho\right) \left(x_1^{-\frac{\alpha}{\rho}} + x_2^{-\frac{\alpha}{\rho}}\right)^{\rho-1} \alpha x_2^{-\frac{\alpha}{\rho}-1} dx_1 \\
&= \exp\left(-\left(\infty^{-\frac{\alpha}{\rho}} + x_2^{-\frac{\alpha}{\rho}}\right)^\rho\right) \left(\infty^{-\frac{\alpha}{\rho}} + x_2^{-\frac{\alpha}{\rho}}\right)^{\rho-1} \alpha x_2^{-\frac{\alpha}{\rho}-1} \\
&- \exp\left(-\left(0^{-\frac{\alpha}{\rho}} + x_2^{-\frac{\alpha}{\rho}}\right)^\rho\right) \left(0^{-\frac{\alpha}{\rho}} + x_2^{-\frac{\alpha}{\rho}}\right)^{\rho-1} \alpha x_2^{-\frac{\alpha}{\rho}-1} \\
&= \exp\left(-x_2^{-\alpha}\right) x_2^{-\frac{\alpha}{\rho}(\rho-1)} \alpha x_2^{-\frac{\alpha}{\rho}-1} - 0 \\
&= \exp\left(-x_2^{-\alpha}\right) \alpha x_2^{-\alpha-1}
\end{aligned}
$$

This is the density of a Frechet distribution with cdf $F_{x_2} = \exp\left(-x_2^{-\alpha}\right)$. Therefore, the marginal distribution is independent of ρ. Note that the previous derivation assumed that $\rho \in (0,1)$.

A.1 Proof of Proposition 2.1

Proof To verify that expression, note that $p_1 = Prob(V_1 > V_2)$. We can rewrite $V_i(x_i, w_i)$ as,

$$
V_i(x_i, w_i) = x_i^{1-\gamma} \int_{y \in \mathbb{Y}} \frac{(e^y w_i)^{1-\gamma}}{1-\gamma} \, dF_i(y) \tag{18}
$$

Relabeling the integral as Ω_i, further rewrite $V_i(x_i, w_i)$ as $x_i^{1-\gamma}\Omega_i$. Note that $V_i(x_i, w_i) < 0$ for any $x_i, w_i > 0$. Since the occupational choice entails picking the maximum between $V_1(x_1, w_1)$ and $V_2(x_2, w_2)$, the choice is equivalent to choosing the minimum between $|V_1(x_1, w_1)|$ and $|V_2(x_2, w_2)|$. Therefore, $Pr(V_1 > V_2) = Pr(|V_1| < |V_2|) = Pr(x_1^{1-\gamma}|\Omega_1| < x_2^{1-\gamma}|\Omega_2|) = Pr(x_1^{1-\gamma} < x_2^{1-\gamma}\frac{|\Omega_2|}{|\Omega_1|})$. Since $\gamma > 1$, [9]

[9] To understand the next equality, note that

$$
F_{x_1}(x_1, x_2) = \frac{d}{dx_1} \int_0^{x_1} \int_0^{x_2} f(z, w) dz dw = \int_0^{x_2} f(z, x_1) dz.
$$

We use standard notation $f(x_1, x_2)$ for the joint probability density function.

$$Pr(V_1 > V_2) = Pr\bigg(x_1(|\Omega_1|/|\Omega_2|)^{1/(1-\gamma)} > x_2\bigg) = \int_0^\infty F_{x_1}(x, x(|\Omega_1|/|\Omega_2|)^{1/(1-\gamma)})dx.$$

The derivative of the joint cumulative density function (2) with respect to x_1 is,

$$F_{x_1}(x_1, x_2) = \exp\left\{-\left[\sum_{i=1}^{2}(T_i^{\alpha/\rho}x_i^{-\alpha/\rho})\right]^{\rho}\right\}\left[\sum_{i=1}^{2}(T_i^{\alpha/\rho}x_i^{-\alpha/\rho})\right]^{\rho-1}\alpha T_1^{\alpha/\rho}x_1^{-\alpha/\rho-1} \tag{19}$$

Substituting for $x_1 = x$ and $x_2 = x\frac{|\Omega_1|}{|\Omega_2|}^{1/(1-\gamma)}$, defining κ_i to be $\frac{|\Omega_1|}{|\Omega_i|}^{1/(1-\gamma)}$ and integrating gives,[10]

$$\int F_{x_1}(x, x(|\Omega_1|/|\Omega_2|)^{1/(1-\gamma)}dx =$$

$$= \int \exp\left\{-\left[\sum_{i=1}^{2}\left(\frac{x\kappa_i}{T_i}\right)^{-\alpha/\rho}\right]^{\rho}\right\}\left[\sum_{i=1}^{2}\left(\frac{x\kappa_i}{T_i}\right)^{-\alpha/\rho}\right]^{\rho-1}\alpha T_1^{\frac{\alpha}{\rho}}x^{-\frac{\alpha}{\rho}-1}dx =$$

$$= \int \exp\left\{-\left[\sum_{i=1}^{2}\left(\frac{x\kappa_i}{T_i}\right)^{-\alpha/\rho}\right]^{\rho}\right\}\left[\sum_{i=1}^{2}\left(\frac{\kappa_i}{T_i}\right)^{-\frac{\alpha}{\rho}}\right]^{\rho-1}\alpha T_1^{\frac{\alpha}{\rho}}x^{\frac{-\alpha}{\rho}(\rho-1)}x^{-\frac{\alpha}{\rho}-1}dx =$$

$$= \left[\sum_{i=1}^{2}\left(\frac{\kappa_i}{T_i}\right)^{-\frac{\alpha}{\rho}}\right]^{-1}T_1^{\frac{\alpha}{\rho}}\int \exp\left\{-\left[\sum_{i=1}^{2}T_i^{\frac{\alpha}{\rho}}\kappa_i^{-\frac{\alpha}{\rho}}x^{-\frac{\alpha}{\rho}}\right]^{\rho}\right\}$$

$$\left[\sum_{i=1}^{2}\left(\frac{\kappa_i}{T_i}\right)^{-\frac{\alpha}{\rho}}\right]^{\rho}\alpha x^{-\alpha-1}dx =$$

$$= \left[\sum_{i=1}^{2}\left(\frac{\kappa_i}{T_i}\right)^{-\frac{\alpha}{\rho}}\right]^{-1}T_1^{\frac{\alpha}{\rho}}\int f(x)dx = T_1^{\frac{\alpha}{\rho}}\left[\sum_{i=1}^{2}\left(\frac{\kappa_i}{T_i}\right)^{-\frac{\alpha}{\rho}}\right]^{-1} \tag{20}$$

Since κ_i equals $\frac{|\Omega_1|}{|\Omega_i|}^{1/(1-\gamma)}$ for $i = 1, 2$, substitution yields,

$$p_1 = \frac{T_1^{\frac{\alpha}{\rho}}|\Omega_1(w_1)|^{\frac{\alpha}{\rho(1-\gamma)}}}{\sum_{i=1}^{2}T_i^{\frac{\alpha}{\rho}}|\Omega_i(w_i)|^{\frac{\alpha}{\rho(1-\gamma)}}} \tag{21}$$

[10] The lower and upper integration limits are understood to be 0 and ∞.

A.2 Proof of Proposition 2.2

Proof We denote by $\tilde{x}_i$ the average ability of a workers who choose occupation i. Given that shocks that workers experience after they have chosen an occupation are of mean equal to one, the amount of efficiency units in occupation i is given by $N_i = p_i \tilde{x}_i$. The distributional assumption on the joint distribution of $\boldsymbol{X} = (x_1, x_2)$ implies that the post-sorting distribution of abilities is also Fréchet.

To derive this result we begin by defining the extreme value $V^* = \min_i \left\{ x_i^{1-\gamma} |\Omega_i| \right\}$. As a result for a given $b > 0$, $Pr(V^* > b) = Pr(x_i^{1-\gamma}|\Omega_i| > b) = Pr(x_i^{1-\gamma} > b/|\Omega_i|)$ for all i, which in turn equals,

$$Pr\left(x_i < \left(\frac{b}{|\Omega_i|} \right)^{1/(1-\gamma)} \right) \text{ for all } i.$$

Using the joint cdf, that probability is given by,

$$\begin{aligned} F\left(\frac{b}{|\Omega_1|}, \frac{b}{|\Omega_2|} \right) &= \exp\left\{ -\left[\sum_{i=1}^{2} T_i^{\frac{\alpha}{\rho}} \left(\frac{b}{|\Omega_i|} \right)^{\frac{-\alpha}{\rho(1-\gamma)}} \right]^{\rho} \right\} = \\ &= \exp\left\{ -\left[\sum_{i=1}^{2} \left(T_i^{\frac{\alpha}{\rho}} |\Omega_i|^{\frac{\alpha}{\rho(1-\gamma)}} b^{\frac{-\alpha}{\rho(1-\gamma)}} \right) \right]^{\rho} \right\} = \\ &= \exp\left\{ -\left[\hat{T}^{\rho} \left(b^{\frac{-\alpha}{\rho(1-\gamma)}} \right)^{\rho} \right] \right\}. \end{aligned} \tag{22}$$

where $\hat{T} = \sum_{i=1}^{2} T_i^{\frac{\alpha}{\rho}} |\Omega_i|^{\frac{\alpha}{\rho(1-\gamma)}}$. Since $Pr(V^* > b) = 1 - Pr(V^* < b)$, the cdf of V^* is given by,

$$Pr(V^* < b) = 1 - \exp\left\{ -\left[\hat{T}^{\rho} b^{-\alpha/(1-\gamma)} \right] \right\}. \tag{23}$$

Note that this is the distribution for the extreme value $V^* = x^{*1-\gamma}|\Omega^*| = \min_i x_i^{1-\gamma}|\Omega_i|$. We are interested in the cdf of x^*, the distribution of abilities post-sorting. To obtain that distribution, note that $Pr(V^* > b) = Pr\big(x^* < \big(\frac{b}{|\Omega^*|}\big)^{1/(1-\gamma)}\big) = Pr(x^* < b^*)$ Using the first term in (22), that probability is given by,

$$Pr(x^* < b^*) = \exp\left\{-\left[\sum_{i=1}^{2} T_i^{\frac{\alpha}{\rho}} \left(\frac{b}{|\Omega_i|}\right)^{\frac{-\alpha}{\rho(1-\gamma)}}\right]^{\rho}\right\} =$$

$$= \exp\left\{-\left[\sum_{i=1}^{2} T_i^{\frac{\alpha}{\rho}} \left(\frac{b}{|\Omega^*|}\right)^{-\frac{\alpha}{\rho(1-\gamma)}} \left(\frac{|\Omega^*|}{|\Omega_i|}\right)^{\frac{-\alpha}{\rho(1-\gamma)}}\right]^{\rho}\right\} =$$

$$= \exp\left\{-\left[\sum_{i=1}^{2} T_i^{\frac{\alpha}{\rho}} \left(\frac{b}{|\Omega^*|}\right)^{\frac{-\alpha}{\rho(1-\gamma)}} \left(\frac{|\Omega^*|}{|\Omega_i|}\right)^{-\alpha/\rho(1-\gamma)}\right]^{\rho}\right\} =$$

$$= \exp\left\{-\left[\sum_{i=1}^{2} T_i^{\frac{\alpha}{\rho}} \left(\frac{|\Omega^*|}{|\Omega_i|}\right)^{\frac{-\alpha}{\rho(1-\gamma)}} b^{*\frac{-\alpha}{\rho}}\right]^{\rho}\right\}$$

$$= \exp\left\{-\left[T^* b^{*\frac{-\alpha}{\rho}}\right]^{\rho}\right\}$$

$$= \exp\left\{-\left[T^{*\frac{-\rho}{\alpha}} b^*\right]^{-\alpha}\right\} \tag{24}$$

where $T_i^* = \sum_{i=1}^{2} T_i^{\frac{\alpha}{\rho}} \left(\frac{|\Omega_i^*|}{|\Omega_i|}\right)^{\frac{-\alpha}{\rho(1-\gamma)}}$.

Equation (24) shows that the distribution of x^*, the ability of workers who have chosen an occupation, is Fréchet. Its shape parameter is equal to α and its scale parameter is $T^{*\frac{\rho}{\alpha}}$. The mean of this distribution is $T^{*\frac{\rho}{\alpha}}\Gamma(1-\frac{1}{\alpha})$.

By letting $|\Omega_i^*| = |\Omega_i|$, we have that

$$T^* = T_i^{\frac{\alpha}{\rho}} / p_i$$

. Thus, the mean of that distribution can be written as $T_i p_i^{\frac{-\rho}{\alpha}} \Gamma(1-\frac{1}{\alpha})$. For occupation 1, it is given by,

$$\tilde{x}_1 = E(x_1) = T_1 p_1^{\frac{-\rho}{\alpha}} \Gamma(1 - 1/\alpha), \tag{25}$$

And for occupation 2 by,

$$\tilde{x}_2 = E(x_2) = T_2 p_2^{\frac{-\rho}{\alpha}} \Gamma(1 - 1/\alpha), \tag{26}$$

Once we have $E(\tilde{x}_1)$ and $E(\tilde{x}_2)$ the result follows:

$$N_i = p_i \tilde{x}_i = T_i p_i^{\frac{\alpha-\rho}{\alpha}} \Gamma(1 - 1/\alpha), \tag{27}$$

A.3 Proof of Proposition 2.3

To begin note that from by combining 2.1 and 2.2, N_i equals

Proof

$$N_i = T_i p_i^{\frac{\alpha-\rho}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) = T_i \left[\frac{T_i^{\frac{\alpha}{\rho}} \Omega_i^{\overline{\frac{\alpha}{\rho(1-\gamma)}}}}{T_1^{\frac{\alpha}{\rho}} \Omega_1^{\overline{\frac{\alpha}{\rho(1-\gamma)}}} + T_2^{\frac{\alpha}{\rho}} \Omega_2^{\overline{\frac{\alpha}{\rho(1-\gamma)}}}}\right]^{\frac{\alpha-\rho}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) =$$

$$T_i \left[\sum_{j=1}^{2} \left(\frac{T_j}{T_i}\right)^{\frac{\alpha}{\rho}} \left(\frac{\Omega_j}{\Omega_i}\right)^{\overline{\frac{\alpha}{\rho(1-\gamma)}}}\right]^{\frac{\rho-\alpha}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{28}$$

Also note that the ratio of the two labor inputs in efficiency units is,

$$\frac{N_1}{N_2} = \frac{T_1}{T_2} \left(\frac{T_1^{\frac{\alpha}{\rho}} \Omega_1^{\overline{\frac{\alpha}{\rho(1-\gamma)}}}}{T_2^{\frac{\alpha}{\rho}} \Omega_2^{\overline{\frac{\alpha}{\rho(1-\gamma)}}}}\right)^{\frac{\alpha-\rho}{\alpha}} = \left(\frac{T_1}{T_2}\right)^{\frac{\alpha}{\rho}} \left(\frac{\Omega_1}{\Omega_2}\right)^{\frac{\alpha-\rho}{\rho(1-\gamma)}} = \left(\frac{T_1}{T_2}\right)^{\frac{\alpha}{\rho}} \left(\frac{w_1^{1-\gamma} E_1}{w_2^{1-\gamma} E_2}\right)^{\frac{\alpha-\rho}{\rho(1-\gamma)}} \tag{29}$$

where $E_i = \mathbb{E}(e^{y_i(1-\gamma)})$. In equilibrium, wages are equal to the marginal products of the two types of labor. Given our aggregate technology,

$$Y = [\theta N_1^{\nu} + (1-\theta) N_2^{\nu}]^{1/\nu} \tag{30}$$

we have that

$$w_1 = 1/\nu \, [\theta N_1^{\nu} + (1-\theta) N_2^{\nu}]^{1/\nu - 1} \, \theta N_1^{\nu-1}$$

and

$$w_1 = 1/\nu \, [\theta N_1^{\nu} + (1-\theta) N_2^{\nu}]^{1/\nu - 1} \, (1-\theta) N_2^{\nu-1}.$$

Thus,

$$\frac{w_1}{w_2} = \left(\frac{\theta}{1-\theta}\right)\left(\frac{N_1}{N_2}\right)^{\nu-1} \tag{31}$$

Substituting (31) into (29), we get

$$\frac{N_1}{N_2} = \left(\frac{T_1}{T_2}\right)^{\frac{\alpha}{\rho}} \left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha-\rho}{\rho}} \left(\frac{N_1}{N_2}\right)^{-(\nu-1)\frac{\rho-\alpha}{\rho}} \left(\frac{E_1}{E_2}\right)^{\frac{\alpha-\rho}{\rho(1-\gamma)}} \tag{32}$$

Simplifying

$$\frac{N_1}{N_2} = \left(\frac{T_1}{T_2}\right)^{\frac{\alpha}{\nu(\rho-\alpha)+\alpha}} \left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha-\rho}{\nu(\rho-\alpha)+\alpha}} \left(\frac{E_1}{E_2}\right)^{\frac{\alpha-\rho}{(\nu(\rho-\alpha)+\alpha)(1-\gamma)}} \tag{33}$$

Note from (28) that N_1 is,

$$N_1 = T_1 \left[1 + \left(\frac{T_2}{T_1}\right)^{\frac{\alpha}{\rho}} \left(\frac{\Omega_2}{\Omega_1}\right)^{\frac{\alpha}{\rho(1-\gamma)}}\right]^{\frac{\rho-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{34}$$

$$= T_1 \left[1 + \left(\frac{T_2}{T_1}\left(\frac{\Omega_1}{\Omega_2}\right)^{\frac{1}{(\gamma-1)}}\right)^{\frac{\alpha}{\rho}}\right]^{\frac{\rho-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{35}$$

and from (29)

$$\frac{N_1}{N_2} = \frac{T_1}{T_2}\left(\frac{T_2}{T_1}\left(\frac{\Omega_1}{\Omega_2}\right)^{\frac{1}{(\gamma-1)}}\right)^{\frac{\rho-\alpha}{\rho}} \tag{36}$$

so that,

$$\frac{T_2}{T_1}\left(\frac{\Omega_1}{\Omega_2}\right)^{\frac{1}{(\gamma-1)}} = \left(\frac{T_2}{T_1}\frac{N_1}{N_2}\right)^{\frac{\rho}{\rho-\alpha}}. \tag{37}$$

Substituting back into (35),

$$N_1 = T_1 \left[1 + \left(\frac{T_2 N_1}{T_1 N_2}\right)^{\frac{\alpha}{\rho-\alpha}}\right]^{\frac{\rho-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) = \left[1 + \left(\frac{T_2}{T_1}\right)^{\frac{\alpha}{\rho-\alpha}} \left(\frac{N_1}{N_2}\right)^{\frac{\alpha}{\rho-\alpha}}\right]^{\frac{\rho-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right)$$

Substituting for the value of the ratio of labor inputs given by (33)

$$N_1 = T_1 \left[1 + \left(\frac{T_2}{T_1} \left(\frac{T_2}{T_1} \right)^{\frac{-\alpha}{\nu(\rho-\alpha)+\alpha}} \left(\frac{\theta}{1-\theta} \right)^{\frac{\alpha-\rho}{\nu(\rho-\alpha)+\alpha}} \left(\frac{E_1}{E_2} \right)^{\frac{\alpha-\rho}{(\nu(\rho-\alpha)+\alpha)(1-\gamma)}} \right)^{\frac{\alpha}{\rho-\alpha}} \right]^{\frac{\rho-\alpha}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{38}$$

Further simplification gives,

$$N_1 = T_1 \left[1 + \left(\frac{T_2}{T_1} \right)^{\frac{\alpha\nu(\rho-\alpha)}{(\nu(\rho-\alpha)+\alpha)(\rho-\alpha)}} \left(\frac{1-\theta}{\theta} \right)^{\frac{\alpha}{\nu(\rho-\alpha)+\alpha}} \left(\frac{E_2}{E_1} \right)^{\frac{\alpha}{(\nu(\rho-\alpha)+\alpha)(1-\gamma)}} \right]^{\frac{\rho-\alpha}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{39}$$

Similarly for N_2 we have,

$$N_2 = T_2 \left[1 + \left(\frac{T_2}{T_1} \right)^{\frac{-\alpha\nu(\rho-\alpha)}{(\nu(\rho-\alpha)+\alpha)(\rho-\alpha)}} \left(\frac{1-\theta}{\theta} \right)^{\frac{-\alpha}{\nu(\rho-\alpha)+\alpha}} \left(\frac{E_2}{E_1} \right)^{\frac{-\alpha}{(\nu(\rho-\alpha)+\alpha)(1-\gamma)}} \right]^{\frac{\rho-\alpha}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{40}$$

By substituting the expressions for N_1 and N_2 into (30) we obtain the competitive equilibrium level of output Y_{CE}.

A.4 Proof of Proposition 4.1

Proof If a firm pays employee in occupation i a wage of w_i, the after-tax wage is $(1-\tau_i)w_i$. Using 2.1 and 2.2, $\frac{N_1}{N_2}$ equals

$$\frac{N_1}{N_2} = \left(\frac{T_1}{T_2} \right)^{\frac{\alpha}{\rho}} \left(\frac{\Omega_1}{\Omega_2} \right)^{\frac{\alpha-\rho}{\rho(1-\gamma)}} = \left(\frac{T_1}{T_2} \right)^{\frac{\alpha}{\rho}} \left(\frac{(1-\tau_1)w_1 E_1^{\frac{1}{1-\gamma}}}{(1-\tau_2)w_2 E_2^{\frac{1}{1-\gamma}}} \right)^{\frac{\alpha-\rho}{\rho}} \tag{41}$$

where the second equality follows from the definition of Ω_i. Because of the aggregate CES technology, wages have to satisfy

$$\frac{w_1}{w_2} = \frac{\theta N_1^{\nu-1}}{(1-\theta)N_2^{\nu-1}} \tag{42}$$

Substituting (42) to the right side of (41) and simplifying gives

$$\frac{N_1}{N_2} = \left(\frac{T_1}{T_2}\right)^{\frac{\alpha}{\alpha-\nu(\alpha-\rho)}} \left(\frac{1-\tau_1}{1-\tau_2}\frac{\theta}{1-\theta}\frac{E_1^{\frac{1}{1-\gamma}}}{E_2^{\frac{1}{1-\gamma}}}\right)^{\frac{\alpha-\rho}{\alpha-\nu(\alpha-\rho)}} \tag{43}$$

From (59) and (60) we have

$$\frac{N_1^{SP}}{N_2^{SP}} = \left(\frac{T_2}{T_1}\right)^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}} \left(\frac{1-\theta}{\theta}\right)^{\frac{\alpha-\rho}{\nu(\alpha-\rho)-\alpha}} \tag{44}$$

Setting the right hand side of (43) equal to the right hand side of (44) gives the expression in the first part of Proposition 4.1.

If we additionally require that government budget is balanced, we have that

$$\tau_1 N_1 w_1 + \tau_2 N_2 w_2 = 0 \tag{45}$$

or, in a more convenient form as

$$\tau_2 = \frac{-\tau_1 N_1 w_1}{N_2 w_2} \tag{46}$$

Substituting in the wage ratio given by equation (42) and evaluating this at $\frac{N_1^{SP}}{N_2^{SP}}$ (given by equation (44)) leads to

$$\tau_2 = -\tau_1 \left(\frac{T_2}{T_1}\right)^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}} \left(\frac{1-\theta}{\theta}\right)^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}} \tag{47}$$

The expression for τ_2 given by (47) can be used to obtain an expression for τ_1 from

$$\frac{1-\tau_1}{1-\tau_2} = \left(\frac{E_2}{E_1}\right)^{\frac{1}{1-\gamma}}$$

Doing so leads to the expression in the latter part of Proposition 4.1 which only depends on the primitives of the model.

A.5 Derivation of the Earnings Premium

From 2.2 we have that

$$p_i = \left(\frac{N_i}{T_i \Gamma(1 - \frac{1}{\alpha})} \right)^{\frac{\alpha}{\alpha - \rho}}. \tag{48}$$

Thus,

$$\frac{p_2}{p_1} = \left(\frac{T_1}{T_2} \right)^{\frac{\alpha}{\alpha - \rho}} \left(\frac{N_2}{N_1} \right)^{\frac{\alpha}{\alpha - \rho}}. \tag{49}$$

Substituting,

$$EP = \frac{w_1}{w_2} \left(\frac{T_1}{T_2} \right)^{\frac{\alpha}{\alpha - \rho}} \left(\frac{N_1}{N_2} \right)^{\frac{-\rho}{\alpha - \rho}}. \tag{50}$$

By using (31) we now have that

$$EP = \frac{\theta}{1 - \theta} \left(\frac{N_1}{N_2} \right)^{\frac{(\rho - \alpha)(\nu - 1) + \rho}{\rho - \alpha}}. \tag{51}$$

From (33)

$$\frac{N_1}{N_2} = \left(\frac{T_2}{T_1} \right)^{\frac{-\alpha}{\rho}} \left(\frac{\Omega_1}{\Omega_2} \right)^{\frac{\alpha - \rho}{\rho(1 - \gamma)}}. \tag{52}$$

Substituting,

$$EP = \frac{w_1}{w_2} \left(\frac{\Omega_1}{\Omega_2} \right)^{\frac{1}{\gamma - 1}}. \tag{53}$$

$$EP = \frac{w_1}{w_2} \left(\frac{w_1^{1-\gamma} E_1}{w_2^{1-\gamma} E_2} \right)^{\frac{1}{\gamma - 1}}. \tag{54}$$

$$EP = \left(\frac{E_1}{E_2} \right)^{\frac{1}{\gamma - 1}}. \tag{55}$$

A.6 The Social Planner's Allocation

We equalize the first order conditions for this problem render (note that the term containing the Γ function cancels out because it is a constant):

$$\theta T_1^{\nu}\left(p_1^{SP}\right)^{\nu\frac{\alpha-\rho}{\alpha}-1} = T_2^{\nu}(1-\theta)\left(p_2^{SP}\right)^{\nu\frac{\alpha-\rho}{\alpha}-1} \tag{56}$$

Since the two masses have to add up to one, we get that

$$p_1^{SP} = \frac{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}}\frac{T_2}{T_1}^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}}}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}}\frac{T_2}{T_1}^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}}+1} \tag{57}$$

and

$$p_2^{SP} = \frac{1}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}}\frac{T_2}{T_1}^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}}+1}. \tag{58}$$

Plugging back into the definition of efficiency units we get the allocation of efficiency units chosen by the social planner:

$$N_1^{SP} = T_1\left[\frac{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}}\frac{T_2}{T_1}^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}}}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}}\frac{T_2}{T_1}^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}}+1}\right]^{\frac{\alpha-\rho}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right) \tag{59}$$

$$N_2^{SP} = T_2\left[\frac{1}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}}\frac{T_2}{T_1}^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}}+1}\right]^{\frac{\alpha-\rho}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right) \tag{60}$$

Given the labor inputs chosen by the planner, the efficient level of output is

$$
\begin{aligned}
Y_{SP} = \Bigg[\theta T_1^{\nu} & \left(\frac{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}} \frac{T_2}{T_1}^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}}}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}} \frac{T_2}{T_1}^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}} + 1} \right)^{\nu \frac{\alpha-\rho}{\alpha}} + \\
& (1-\theta) T_1^{\nu} \left(\frac{1}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-\rho)-\alpha}} \frac{T_2}{T_1}^{\frac{\alpha\nu}{\nu(\alpha-\rho)-\alpha}} + 1} \right)^{\nu \frac{\alpha-\rho}{\alpha}} \Bigg]^{1/\nu} \Gamma\left(1 - \frac{1}{\alpha}\right)
\end{aligned} \tag{61}
$$